A Practical Approach of Complex Dual Tree DWT for Image Quality Improvement and De-noising

Rudra Pratap Singh Chauhan[1], Sanjay Singh[2], Sanjeev Kumar Shah[3]
(Deptt.-ECE, Uttaranchal Institute of Technology, Dehradun, Uttarakhand, India,
(Deptt.-ECE, Uttaranchal Institute of Technology, Dehradun, Uttarakhand, India,
(Deptt.-ECE, Uttaranchal Institute of Technology, Dehradun, Uttarakhand, India,

ABSTRACT
It is pragmatic that for many natural signals and images, the wavelet transform is a more effective and powerful signal processing tool that provide an alternative to conventional signal transforms such as the Fourier transform. The wavelet transform provides a multi resolution representation using a set of analyzing functions that are dilation and translations of a few functions. Although the standard DWT is powerful tool, it has three major limitations (Shift Sensitivity, Poor Directionality, and Absence of Phase information) that undermines its application for certain signal and image processing tasks. The initial motivation behind the work to use Dual-tree complex-valued DWT which will overcome the limitations of standard DWT. This paper, proposes a new simple non Gaussian bivariate probability distribution function to model statistics of wavelet coefficients of images. The model captures the dependence between a wavelet coefficient and its parent. The new bivariate shrinkage function outperforms the method proposed by the previous authors for image de-noising and also give significant improvement in the value of PSNR over RMSE as given by the other classical methods.

KEYWORDS- Bivariate Shrinkage Function. Complex DWT, De-noising, 1-D DWT, 2-D DWT.

1. INTRODUCTION
Many scientific experiments results in a datasets corrupted with noise, either because of the inadequate data acquisition process, or because of environmental effects. A first preprocessing step in analyzing such datasets is de-noising, that is, removing the unknown signal of interest from the available noisy images. There are several approaches to de-noise images. Despite similar visual effects, there are subtle differences between de-noising, de-blurring, smoothing and restoration.

Generally smoothing removes high frequency and retain low frequency(with blurring), de-blurring increases the sharpness signal features by boosting the high frequencies, whereas de-noising tries to remove whatever noise is present regardless of the spectral content of a noisy image. Restoration is kind of de-noising that tries to retrieve the original image with optimal balancing of de-blurring and smoothing. Traditional smoothing filters such as Mean, Median and Gaussian filters are linear operators normally employed in spatial domain, which smooth the signals with blurring effects.

For many natural images, the wavelet transform is more efficient and effective tool because it is based on perfect reconstruction two-channel (analysis/synthesis) filter banks. The DWT consists of recursively applying a 2-channel filter banks, the successive decomposition is performed only on the low-pass output. The wavelet transform comes in several forms. The critically sampled form of the wavelet transform provides the most compact representation, however, it has several limitations. For example, it lacks the shift-invariance property, and in multiple dimensions it does a poor job of distinguishing orientations, which is important in image processing. For these reasons, it turns out that for some applications improvements can be obtained by using an expansive wavelet transform in place of critically-sampled one. An expensive transform is one that converts an N-point signal into M coefficient with M > N. There are several kinds of expansive DWTs; here we describe and provide an implementation of the dual-tree complex DWT.

This paper presents the concept of complex dual tree DWT[5] to de-noise the digital images by using soft thresholding algorithm with new shrinkage function. This transform is shift invariant and is oriented in 2D. The 2D dual tree wavelet transform produces six sub bands at each scale, each of which is strongly oriented at distinct angles. The paper is organize as follows: Section 2 involves Separable DWT. The Complex Dual Tree DWT is discussed in Section 3 & Section 4 deals with bivariate shrinkage functions. Section 5 gives the general method involves in image de-noising. Section 6 deals with results & discussions & the conclusion are given in Section 7.

2. Separable DWT
2.1 1-D Discrete Wavelet Transform
The analysis filter bank decomposes the input signal x(n) into two sub band signals, c(n) and d(n). The signal c(n) represents the low frequency part of x(n), while the signal d(n) represents the high frequency part of x(n). We denote the low pass filter by af1(analysis filter 1) and the high pass filter by af2(synthesis filter 2). As depicted in figure(1), the output of each filter is then down sampled by 2 to obtain the two sub band signals c(n) & d(n).

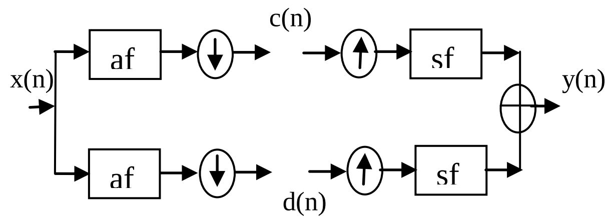

Fig.1: Separable DWT(Analysis) & (Synthesis) Filter Bank

The Synthesis filter bank combines the two sub band signals c(n) & d(n) to obtain a single signal y(n). The synthesis filter bank up-samples each of the two sub band signals. The signals are then filtered using a low pass and high pass filter. We denote the low pass filter by sf1(synthesis filter 1) and the high pass filter by sf2(synthesis filter 2). The signals are then added together to obtain the signal y(n). If the four filters are designed so as to guarantee that the output signal y(n) equals the input signal x(n), then the filters are said to satisfy the perfect reconstruction condition.

2.2 2-D Discrete Wavelet Transform

image-processing applications requires two-dimensional implementation of wavelet transform. Implementation of 2-D DWT is also referred to as 'multidimensional' wavelet transform in literature. In the 2D case, the 1D analysis filter bank is first applied to the columns of the image and then applied to the rows. If the image has N1 rows and N2 columns, then after applying the 1D analysis filter bank to each column we have two sub band images, each having N1/2 rows and N2 columns; after applying the 1D analysis filter bank to each row of both of the two sub band images, four sub band images are obtained, each having N1/2 rows & N2/2 columns. This is depicted in figure (2) given below. The 2D synthesis filter bank combines the four sub band images to obtain the original image of size N1 by N2 [4][5].

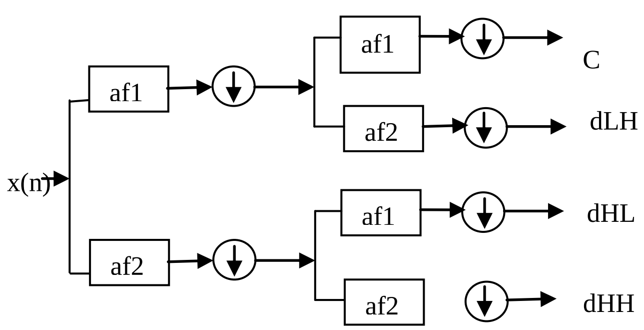

Fig.(2): One stage in multi-resolution wavelet decomposition of an image x(n)

The 2D analysis filter bank is implemented with the Matlab function afb.m. This function calls a sub-function, afb2D_A.m, which applies the 1D analysis filter bank along one dimension only either along the columns or along the rows. The function afb2D.m returns two variables: 'lo' is the low-pass sub band image; 'hi' is a cell array containing the 3 other sub band images. The 2D synthesis filter bank is similarly implemented with the commands sfb2D.m and sfb2D_A.m. The 2D DWT of a signal x(n) is implemented by iterating the 2D analysis filter bank on the low-pass sub band image. In this case, at each scale there are three sub bands instead on one. The function, dwt2D.m, computes the J-scale 2D DWT w of an image x(n) by repeatedly calling afb2D.m. Again, w is a Matlab cell array; for j = 1..j, d = 1..3, w{j}{d} is one of the three sub band images produced images produced at stage j. w{j+1} is the low-pass sub band image produced at the last stage. The function idwt2D.m computes the inverse 2D DWT.

3. Complex 2D Dual –Tree DWT:

To preserve the phase information of image for some applications DWT fails, this limitation of discrete wavelet transform can be overcome by using an expensive wavelet transform rather than a critically sampled one. There are several kinds of expensive DWT's; here the dual tree CDWT is described. The dual tree complex DWT[1][2] of a signal x(n) is implemented using two critically sampled DWT's in parallel on the same data, as depicted in the figure(3).

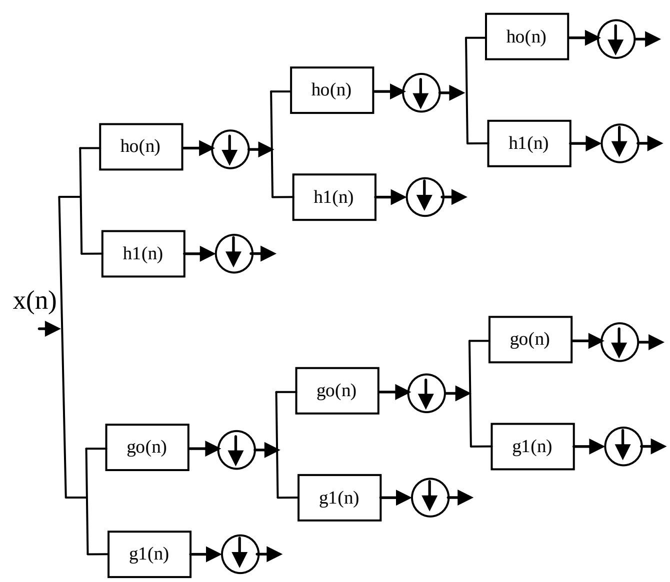

Fig.(3): Dual Tree Complex DWT

The transform is 2-times expansive because for an N-point signal it gives 2N DWT coefficients. To overcome the limitations of DWT, here filters are designed is a specific way. The complex 2D dual tree DWT gives rise to wavelets in six distinct directions, however, in this case there are two wavelets in each direction as shown in the above figure (3). In each direction, one of the two wavelets can be interpreted as the real part of a complex-valued 2D wavelet, while the other wavelet can be interpreted as the imaginary part of a complex-valued 2D wavelet. Because the complex version has twice as many wavelets as the real version of the transform, the complex version is 4-times expansive. The complex 2D dual-tree is implemented as four critically-sampled separable 2D DWTs operating in parallel. However, different filter sets are used along the rows and columns. As in the real case, the sum and difference of sub band images is performed to obtain the oriented wavelets. The complex 2D dual-tree DWT of an image x(n) is computed by the Matlab function, cplxdual.m. The wavelet coefficients w are stored as a cell array. For j = 1..J, p = 1..2, k = 1..2,, d = 1..3, w{j}{p}{k}{d} are the wavelet coefficients produced at scale j and orientation (k,d). With p = 1 we get the real part, with p = 2 we get the imaginary part. The image x(n) is recovered from w using the inverse transform, implemented by icplxdual2D.m

The filter banks in dual-tree complex DWTs discussed above used for image de-noising is nearly shift invariant, in contrast with the critically sampled DWT. The dual-tree complex DWT can be used to implement 2D wavelet transform where each wavelet is oriented, which is especially useful for image processing. The dual-tree complex DWT outperforms the separable DWT for applications like image de-noising.

4. Bivariate Shrinkage Function

To model the statistics of wavelet coefficients of images, a new simple non-Gaussian bivariate probability distribution function[3][6-9] is proposed in this paper. The model captures the dependence between a wavelet coefficient and its parent. Using Bayesian estimation Theory, this model is derived, which generalizes the soft thresholding approach. The new shrinkage function, which depends on both the coefficient and its parent, yields improved results for wavelet based image de-noising.

Let w2 represents the parent of w1(w2 is the wavelet coefficient at the same spatial position as w1, but at the next coarser scale). Then,

$$y = w + n \text{ ------------------ (1)}$$

When w = (w1,w2), y = (y1,y2) and (n1,n2). The noise values n1, n2 are zero mean Gaussian. Based on the empirical histograms, the non-Gaussian Bivariate PDF is given by,

$$p_w(w) = \frac{3}{2\pi\sigma^2} \cdot exp\left(-\frac{\sqrt{3}}{\sigma}\sqrt{w_1^2+w_2^2}\right) \text{------------(2)}$$

With this PDF , w1 and w2 are uncorrelated, but not independent. The MAP estimator of w1 yields the following bivariate shrinkage function:

$$\hat{w} = \frac{\left(\sqrt{y_1^2 + y_2^2} - \frac{\sqrt{3}\sigma_n^2}{\sigma}\right)}{\sqrt{y_1^2 + y_2^2}} \cdot y_1 \text{ ---- (3)}$$

For this bivariate shrinkage function, the smaller the parent value, the greater the shrinkage. This is consistent with other models, but here is derived using a Bayesian estimation approach beginning with the new bivariate non-Gaussian model.

4.1 De-noising Algorithm

The implementation of the de-noising algorithm is similar to the separable DWT case. Except that there are slight modifications since we apply the bivariate shrinkage rule to the magnitudes of the complex coefficients. They are nearly shift invariant, i.e. small signal shift do not affect the magnitudes, however, they do affect the real and imaginary parts. Therefore magnitude information is more reliable than either real or imaginary parts. The algorithm does not affect the angle, i.e. we keep it as it is. The function main_dtdwt.m loads the noisy image, calls the de-noising routine and calculate the PSNR value of de-noised image.

After loading the input image by the “main_dtdwt.m” function, the calculations for the local adaptive image de-noising are done by a Matlab function denoising_dtdwt.m. This function calls several sub-functions. The implementation can be summarized as:

1. Set the window size. The signal variance of a coefficient will be estimated using neighboring coefficients in a rectangular region with this window size.
2. Set how many stages will be used for the wavelet transform.
3. Extend the noisy image. The noisy image will be extended using symmetric extension in order to improve the boundary problem with the function symextend.m.
4. Calculate the forward dual-tree DWT using cplxdual2D.m.
5. Estimate the noise variance. The noise variance will be calculated using the robust median estimator.
6. Process each sub band separately in a loop. First the real and imaginary parts of the coefficients and the corresponding parent matrices are prepared for each sub band, and the matrices corresponding to the real and imaginary parts of the parent matrix are expanded using function expand.m in order to make the matrix size the same as the coefficient matrix.
7. Estimate the signal variance and the threshold value: The signal variance for each coefficient is estimated using the window size and the threshold value for each coefficient will be calculated and stored in a matrix with the same size as the coefficient matrix.
8. Estimate the magnitude of the complex coefficients. The coefficient will be estimated using the magnitudes of the complex coefficient, its parent and the threshold value with matlab function bishrink.m.
9. Calculate the inverse wavelet transform using matlab function icplxdual2D.m.
10. Extract the image. The necessary part of the final image is extracted in order to reverse the symmetrical extension.

5. General Method for Image De-noising

The classical methods used by the previous authors for image de-noising employs soft thresholding image. The new local adaptive de-noising algorithm via bishrinkage function are as follows:

5.1 Soft Thresholding

One techniques for de-noising is wavelet thresholding. When we decompose data using the wavelet transform, we use filters that act as averaging filters, and others that produce details. Some of the resulting coefficients correspond to details in the data set (high frequency sub-bands). If the details are small, they might be omitted without substantially affecting the main features of the data set. The idea of thresholding is to set all high frequency sub bands coefficients that are less than a particular threshold to zero. These coefficients are used in an inverse transformation to reconstruct the data set.

The Matlab program for soft thresholding has two parameters, one for noise signal and the other for threshold point. A sample noise signal with dimension 512 x 512 is taken. We first take the forward DWT over 4 scales (J = 4). Then a de-noising method called soft thresholding is applied to wavelet coefficients through all scales and sub bands. Function soft sets coefficients with values less than the threshold (T) to 0, then subtracts T from the non-zero coefficients. After soft thresholding, we take inverse wavelet transform.

5.2 Peak Signal to Noise Ratio (PSNR)

Peak Signal-to-Noise Ratio is given by the following expression:

$$\mathbf{PSNR} = 10 \log_{10} \left(\frac{255}{\mathrm{RMSE}} \right)$$

Where,

RMSE – Root Mean Square Error

5.3 Proposed Method: Bivariate Shrinkage Function + Local Adaptive de-noising Algorithm

The proposed method which uses both bivariate Shrinkage Function together with Local adaptive de-noising algorithm is described here. The algorithm of the proposed method is summarized as follows:

1. Take the natural image as input and denote it as x(n).
2. Add noise into the x(n). Let the noisy image be $x_1(n)$.
3. Take the Forward dual-tree complex DWT on noisy image $x_1(n)$. Use analysis filter bank.
4. Compute Bivariate Shrinkage Function.
5. Compute Local Adaptive De-noising algorithm.
6. Compute Inverse dual-tree complex DWT. Use synthesis filter bank.
7. Get the de-noised image.

6. Results and Discussion

From the resulting images, it is clear that the Complex 2D dual-tree method removes more noise signal than separable 2D method does. The 2D dual-tree method outperforms separable method. This can be proved by the RMSE Vs PSNR points.

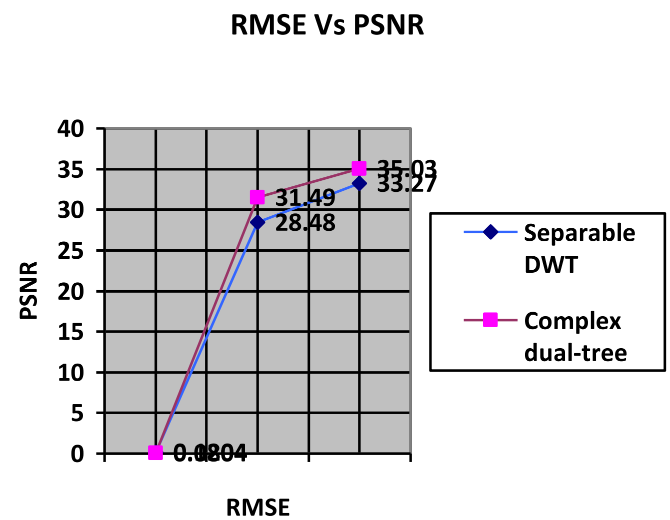

Fig.(4): RMS Error Vs Peak Signal-to- Noise Ratio

6.1 Results via Soft Thresholding

Original Image **Noisy Image**

Separable DWT **Complex Dual-Tree DWT**

Fig.(5): Output Results via Soft Thresholding

6.2 Results via Bivariate Shrinkage Function

Separable DWT Complex Dual-Tree DWT

Fig.(6): Output Results via Bivariate Shrinkage Function

The de-noised image obtained using soft thresholding has a PSNR of 33.27 dB. The de-noised image obtained using bivariate shrinkage function has a PSNR of 35.03 dB. Thus the local adaptive thresholding algorithm via bivariate function gives better performance over the classical method. The values are tabulated as follows:

Types of Method	RMSE	Soft Thresholding in dB	Bivariate Shrinkage in Db
		PSNR[dB]	PSNR[dB]
Separable DWT	0.1204	28.48	33.27
Complex dual-tree DWT	0.0804	31.49	35.03

Table1: Root Mean Square Error (RMSE) & Peak Signal-to-Noise Ratio (PSNR) Values

7. Conclusion and Future Scope

In this paper, a new bivariate PDF is proposed for wavelet coefficients from which a new bivariate shrinkage function is derived. The proposed PDF is a Laplacian bivariate PDF. This new rule maintains the simplicity, efficiency and classical soft thresholding approach. The result is simulated on Matlab 7.0.1 environment. The Simulation results of Aishwarya Rai for classical Separable DWT and Complex Dual-Tree DWT are shown in above figures(5) and (6). With these results it is clear that the proposed method gives significant improvement in terms of image quality and preserve the useful information from the original image. These properties are important for many applications in image processing. In future, this work can be extended by using different types of wavelets for different values of noise variance also.

References

[1] I. Bayram and I.W. Selesnik, "On the dual-tree complex wavelet packet and M-band transforms" IEEE Trans. On Signal Processing, 56(6):2298-2310, June 2008.

[2] I. W. Selesnick, R. G. Baraniuk, and N. Kingsbury, "The dual-tree complex wavelet transform- A coherent framework for multiscale signal and image processing", IEEE Signal Processing Magazine, 22(6): 123-15, November 2005.

[3] R. Gomathi & S. Sevakumaran, "A Bivariate Shrinkage Function for Complex Dual-Tree DWT based Image De-noising", in Proc. ICWAMS-2006, Bucharest, Romania, October 16-18, 2006.

[4] I. W. Selesnick, "The Double Density DWT," in wavelets in signal and Image analysis: From Theory to Practice, A. Petrosian and F.G. Meyer, Eds. Boston, MA: Kluwer, 2001.

[5] I. W. Selesnick, "The Double Density Dual-Tree DWT," IEEE Trans. On Signal Processing, 52(5): 1304-1314, May 2004.

[6] L. Sendur and I. W. Selesnik, "Bivariate Shrinkage with local variance estimation," IEEE Signal Processing Letters, 9(12), December 2002.

[7] L. Sendur and I. W. Selesnik, "Bivariate Shrinkage Functions for wavelet-based de-noising exploiting interscale dependency," IEEE Trans. On Signal Processing, 50(11): 2744-2756, November 2002.

[8] L. Sendur, I. W. Selesnick, " A bivariate shrinkage function for wavelet-based denoising", IEEE International Conference on Acoustics, Speech, and Signal Processing, 2002.

[8] N. G. Kingsbury, " Complex wavelets for shift invariant analysis and filtering of signals," Applied and Computational Harmonic Analysis, 10(3): 234-253, May 2002.

[9] L. Sendur, I. W. Selesnick, " Sub band adaptive image de-noising via bivariate shrinkage", IEEE International Conference on Image Processing (ICIP), 2002.